

Financial Frictions, Investment and Tobin's q *

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Abstract

We develop a model of investment with financial constraints and use it to investigate the relation between investment and Tobin's q . A firm is financed partly by insiders, who control its assets, and partly by outside investors. When their wealth is scarce, insiders earn a rate of return higher than the market rate of return, i.e., they receive a quasi-rent on invested capital. This rent is priced into the value of the firm, so Tobin's q is driven by two forces: changes in the value of invested capital, and changes in the value of the insiders' future rents per unit of capital. This weakens the correlation between q and investment, relative to the frictionless benchmark. We present a calibrated version of the model, which, due to this effect, generates realistic correlations between investment, q , and cash flow.

Keywords: Financial constraints, optimal financial contracts, investment, Tobin's q , limited enforcement.

JEL codes: E22, E30, E44, G30.

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1 Introduction

The standard model of investment with convex adjustment costs predicts that movements in the investment rate should be entirely explained by changes in Tobin's q . This prediction has generally been rejected in empirical studies, which show that cash flow and other measures of current profitability have a strong predictive power for investment, after controlling for q . These results have been interpreted as evidence of financial frictions which make investment more sensitive to internal sources of finance, for which cash-flow is a natural proxy, than to expected profitability, captured by q . Starting with Fazzari, Hubbard, and Petersen (1988), this interpretation has been pursued and developed by a vast empirical literature.¹ At the same time, a literature which goes back to Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997), and Bernanke, Gertler and Gilchrist (2000), has developed models of investment with financial constraints which are incorporated into full dynamic general equilibrium models and used to study the aggregate implications of financial frictions.² These models capture the financial constraint by focusing on some basic informational or enforcement friction which affects the agents who run the firms in the economy. In this paper, we study the implications of a model of this type for the q theory of investment. That is, we ask whether a simple microfounded model of financial frictions can deliver realistic correlations between investment, q , and cash-flow. This exercise provides a basic test of consistency for this class of models.

We start from a setup with firms controlled by “insiders,” who can be interpreted as the entrepreneur, the manager, or the controlling shareholder. We assume that the insider has the ability to partially divert the assets of the firm and, if he does so, he is punished by losing control of the firm. In this setup, we characterize the optimal long-term financial contract between the insider and the outside investors.

Aside from the enforcement friction, our model is virtually identical to the classic Hayashi (1982) model. In particular, it features convex adjustment costs and constant returns to scale. This allows us to identify in a clean way the effect of the financial friction on the equilibrium behavior of investment and q . As in Hayashi (1982), we make a distinction between marginal q , which captures the marginal value of new investment, and average q , which is the ratio of the total value of the firm to the replacement value of its capital stock. Average q is observable and, thus, is the notion of q typically used in empirical studies.

On the analytical side, we show that the financial constraint introduces a positive wedge between marginal q and average q . The wedge between average and marginal q reflects the tension between the future profitability of investment and the availability of internal funds in the short run. On the quantitative side, we use a calibrated version of the model to

¹See Hubbard (1998) for a survey and references below.

²A non-exhaustive list of recent applications of “financial accelerator” models includes Christiano, Motto, and Rostagno (2003), Iacoviello (2005), Gertler, Gilchrist, and Natalucci (2007), Faia and Monacelli (2007).

show that this wedge varies over time, breaking the one-to-one correspondence between investment and average q which holds in the frictionless model. We then simulate the model and run standard investment regressions on the simulated data. From this exercise we obtain coefficients on q and cash flow which are in line with those found in empirical regressions. We conclude that our simple model is able to replicate the basic correlation patterns between investment, asset prices, and cash-flow.

To assess the role of the financial friction, we compare the quantitative implications of our model with those of the frictionless benchmark. In the frictionless benchmark, the coefficient on q in the investment regression is equal to the inverse of the coefficient on the quadratic term in the adjustment cost function. The presence of the financial friction reduces this coefficient by a factor of six while, at the same time, generating a large positive coefficient on cash flow.

A growing number of papers uses recursive methods to characterize optimal dynamic financial contracts in environments with different forms of contractual frictions (DeMarzo and Fishman, 2003, Quadrini, 2004, Clementi and Hopenhayn, 2006, DeMarzo and Sannikov, 2006, and Atkeson and Cole, 2005). The limited enforcement friction in this paper makes it closer to the models in Albuquerque and Hopenhayn (2004) and Cooley, Marimon and Quadrini (2004). Within this literature, Biais, Mariotti, Plantin and Rochet (2007) look more closely at the implications of the theory for asset pricing. In particular, they find a set of securities that implements the optimal contract and then study the stochastic behavior of the prices of these securities. Here, our objective is to examine the model's implication for q theory, therefore we simply focus on the total value of the firm, which includes the value of all the claims held by insiders and outsiders. A distinctive feature of our approach is that we make heavy use of the assumption of constant returns to scale, so that optimal contracts take a simple linear form. This makes aggregation straightforward and makes our model very easy to incorporate into a general equilibrium environment. In this sense, the model retains the simplicity of a representative agent model, while allowing for rich dynamics of net worth, profits and investment.

The idea of looking at the statistical implications of a simulated model to understand the empirical correlation between investment and q goes back to Sargent (1980). Recently, Gomes (2001), Cooper and Ejarque (2001, 2003) and Abel and Eberly (2004, 2005) have followed this route, introducing both financial frictions and decreasing returns and market power to match the existing empirical evidence.³ The conclusion one can derive from this set of papers is that decreasing returns and market power help to generate realistic correlations, while financial frictions do not. In particular, Gomes (2001) and Cooper and Ejarque (2003) obtain negative results which stand in contrast to the results obtained here. In their simulated economies with financial frictions Tobin's q still explains most of

³See Schiantarelli and Georgoutsos (1990) for an early study of q theory in a model where firms have market power.

the variability in investment, and cash flow does not provide any additional explanatory power.⁴ The main difference between our approach and their approach is the modeling of the financial constraint. They introduce a constraint on the flow of outside finance that can be issued each period. Here instead, we explicitly model a contractual imperfection and solve for the optimal long-term contract. This adds a state variable to the problem, namely the stock of existing liabilities of the firm, thus generating slow-moving dynamics in the gap between internal funds and the desired level of investment. These dynamics account for the empirical disconnect between investment and q and explain why our model of financial frictions is better able to replicate observed correlations. On the other hand, there are some parallels between our approach and the approach based on decreasing returns and market power, in particular with the “growth options” mechanism emphasized in Abel and Eberly (2005). Both approaches imply that movements in q can reflect changes in future rents that are unrelated to current investment. In our paper these rents are not due to market power, but to the scarcity of entrepreneurial wealth, which evolves endogenously.

Following Fazzari, Hubbard and Petersen (1988) there has been a large empirical literature exploring the relation between investment and q using firm level data. The majority of these papers find small coefficients on q and positive and significant coefficients on cash flow and other variables describing the current financial condition of a firm. This result has been ascribed to measurement error in q , possibly caused by non-fundamental stock market movements.⁵ Measurement error would reduce the explanatory power of q , and cash flow would then appear as significant, given that it is a good predictor of future profits. Gilchrist and Himmelberg (1995) show that this is insufficient to explain the failure of q theory in investment regressions.⁶ They replace the value of q obtained by financial market prices with a measure of “fundamental q ” (which employs current cash flow as a predictor of future profits), and they show that current cash flow retains its independent explanatory power. Finally, Hennessy and Whited (2007) build a rich structural model of firms’ investment with financial frictions, which is then estimated by simulated method of moments on firm-level data. They find that the financial constraint plays an important role in explaining observed firms’ behavior.⁷ The evidence in this literature provides the starting motivation for our exercise. In an extension of the model (Section 4) we introduce firm-level heterogeneity and further explore the connection between our model and panel data evidence.

The paper is organized as follows. Section 2 presents the model, the derivation of the

⁴See also Moyen (2004).

⁵The debate is open whether non-fundamental movements in q should affect investment or not. See Chirinko and Schaller (2001), Gilchrist, Himmelberg and Huberman (2005) and Panageas (2005).

⁶See Erickson and Whited (2000) and Bond and Cummins (2001) for a contrarian view. See Rauh (2006) for recent evidence in favor of the financial frictions interpretation.

⁷In their model, due to the complexity of the estimation task, the financial friction is introduced in a relatively “reduced form,” by assuming that there are some transaction costs associated to the issuance of new equity or new debt, as in Cooper and Ejarque (2003) and Gomes (2001). The difference in results, relative to these papers, appears due to the fact that Hennessy and Whited (2007) also match the behavior of a number of financial variables.

optimal contract, and the equilibrium analysis. Section 3 contains the calibration and simulation results. In Section 4 we extend the model to allow for firm-level heterogeneity. Section 5 concludes. All proofs not in the text are in the appendix.

2 The Model

2.1 The environment

Consider an economy populated by two groups of agents of equal mass, consumers and entrepreneurs. Consumers are infinitely lived and have a fixed endowment of labor l_C , which they supply inelastically on the labor market at the wage w_t . Consumers are risk neutral and have a discount factor β_C . Entrepreneurs have finite lives, with a constant probability of death γ . Each period, a fraction γ of entrepreneurs is replaced by an equal mass of newly born entrepreneurs. Entrepreneurs begin life with no capital and have a labor endowment l_E in the first period of life, which gives them an initial wealth $w_t l_E$. Entrepreneurs are also risk neutral, with a discount factor $\beta_E < \beta_C$. The last assumption, together with the assumption of finitely lived entrepreneurs, is needed to ensure the existence of a steady state with a binding financial constraint. We normalize total labor supply to one, that is, $l_C + \gamma l_E = 1$.

Starting in their second period of life, entrepreneurs have access to the constant-returns-to-scale technology $A_t F(k_t, l_t)$, where k_t is the stock of capital installed in period $t - 1$, and l_t is labor hired on the labor market. The productivity A_t is equal across entrepreneurs and follows the stationary stochastic process $A_t = \Gamma(A_{t-1}, \epsilon_t)$, where ϵ_t is an i.i.d. shock drawn from the discrete p.d.f. $\pi(\epsilon_t)$. We normalize the unconditional mean of A_t to 1.

Investment in new capital is subject to convex adjustment costs, which we now describe. At the end of each period, after production has taken place, capital depreciates and the entrepreneur has access to k_t units of used capital. Then, he can trade capital on the used capital market, at the price q_t^o , and choose to hold on to k_t^o units of used capital, where k_t^o may be different from k_t . Next, the entrepreneur chooses the capital stock for next period k_{t+1} , and pays $G(k_{t+1}, k_t^o)$, which includes both the cost of acquiring new capital and the installation (or uninstallation) costs. The function $G(k_{t+1}, k_t^o)$ is increasing and convex in k_{t+1} , decreasing in k_t^o , homogeneous of degree one, and satisfies $\partial G(k_{t+1}, k_t^o) / \partial k_{t+1} = 1$ if $k_{t+1} = k_t^o$.

An entrepreneur born at date t_0 finances his current and future investment by issuing a long-term financial contract, specifying a sequence of state-contingent transfers (which can be positive or negative) from the entrepreneur to the outside investors, $\{d_t\}_{t=t_0}^\infty$. In period $t = t_0$, his budget constraint is

$$c_t^E + G(k_{t+1}, k_t^o) + q_t^o k_t^o \leq w_t l_E - d_t.$$

The entrepreneur uses his initial wealth to consume and to acquire used capital and transform it into capital ready for use in $t_0 + 1$. Furthermore, he can increase his consumption and investment by borrowing from consumers, i.e., choosing a negative value for d_{t_0} . In the remaining periods, the budget constraint is

$$c_t^E + G(k_{t+1}, k_t^o) + q_t^o(k_t^o - k_t) \leq A_t F(k_t, l_t) - w_t l_t - d_t.$$

He uses current revenues, net of labor costs and financial payments, to finance consumption and investment. At the beginning of each period t , the entrepreneur learns whether that is his last period of activity. Therefore, in the last period, he liquidates all the capital k_t and consumes the receipts, setting

$$c_t^E = A_t F(k_t, l_t) - w_t l_t + q_t^o k_t - d_t.$$

From then on, the payments d_t are set to zero.

Financial contracts are subject to *limited enforcement*. The entrepreneur controls the firm's assets k_t and can, in each period, run away, diverting a fraction $(1 - \theta)$ of them. If he does so, he re-enters the financial market as if he was a young entrepreneur, with initial wealth given by the value of the diverted assets, and zero liabilities. That is, the only punishment for a defaulting entrepreneur is the loss of a fraction θ of the firm's assets.⁸ Aside from limited enforcement no other imperfections are present, in particular, financial contracts are allowed to be fully state-contingent.

2.2 Recursive competitive equilibrium

We will focus our attention on recursive equilibria where the economy's dynamics are fully characterized by the vector of aggregate state variables $X_t \equiv (A_t, K_t, B_t)$, where K_t is the aggregate capital stock and B_t denotes the aggregate liabilities of the entrepreneurs, to be defined in a moment. In the equilibria considered, consumers always have positive consumption. Therefore, the market discount factor is equal to their discount factor, β_C , and the net present value of the liabilities of an individual entrepreneur can be written as

$$b_t = \mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta_C^s d_{t+s} \right].$$

The variable B_t is equal to the economy-wide aggregate of these liabilities.

A recursive competitive equilibrium is defined by law of motions for the endogenous

⁸Here, we just take this as an institutional assumption. For a microfoundation, we could assume that defaulting entrepreneurs are indistinguishable from young entrepreneurs. However, this would require addressing a number of informational issues, which would considerably complicate the analysis.

state variables:

$$\begin{aligned} K_t &= \mathcal{K}(X_{t-1}), \\ B_t &= \mathcal{B}(X_{t-1}, \epsilon_t), \end{aligned}$$

and by two maps, $w(X_t)$ and $q^o(X_t)$, which give the market prices as a function of the current state. Given these four objects, we can derive the optimal individual behavior of the entrepreneurs. The quadruple $\mathcal{K}, \mathcal{B}, w(\cdot)$ and $q^o(\cdot)$ forms a recursive competitive equilibrium if: (i) the entrepreneurs' optimal behavior is consistent with the law of motions \mathcal{K} and \mathcal{B} , and (ii) the labor and used capital market clear. In the next two subsections, we first characterize entrepreneurs' decisions, and then aggregate and check market clearing. We use

$$X_t = H(X_{t-1}, \epsilon_t),$$

to denote in a compact way the law of motion for X_t derived from the laws of motion Γ, \mathcal{K} , and \mathcal{B} .

2.3 Optimal financial contracts

Let us consider first the optimization problem of the individual entrepreneur. Exploiting the assumption of constant returns to scale, we will show that the individual problem is linear. This property will greatly simplify aggregation.

We describe the problem in recursive form, dropping time subscripts. Consider a continuing entrepreneur, in state X , who controls a firm with capital k and outstanding liabilities b . Let $V(k, b, X)$ denote his end-of-period expected utility, computed after production takes place and assuming that the entrepreneur chooses not to default in the current period. The entrepreneur takes as given the law of motion for the aggregate state X and the pricing functions $w(X)$ and $q^o(X)$.

The budget constraint takes the form

$$c^E + G(k', k^o) + q^o(X)k^o \leq AF(k, l) - w(X)l + q^o(X)k - d.$$

Lemma 1 allows us to rewrite it as

$$c^E + q^m(X)k' \leq R(X)k - d, \tag{1}$$

where $q^m(X)$ is the shadow price of the new capital k' , and $R(X)$ is the (gross) return per unit of capital, on the installed capital k .

Lemma 1 *Given the prices $w(X)$ and $q^o(X)$, there are two functions $q^m(X)$ and $R(X)$*

that satisfy the following conditions for any k' and k ,

$$\begin{aligned} q^m(X) k' &= \min_{k^o} \{G(k', k^o) + q^o(X) k^o\}, \\ R(X) k &= \max_l \{AF(k, l) - w(X) l\} + q^o(X) k. \end{aligned}$$

This lemma exploits the assumption of constant returns to show that $q^m(X)$ and $R(X)$ are independent of the current and future capital stocks, k and k' , and only depend on the prices $w(X)$ and $q^o(X)$. The variable $q^m(X)$ is equal to marginal q in our model, and will be discussed in detail below.

A continuing entrepreneur can satisfy his existing liabilities b either by repaying now or by promising future repayments. Let $b'(\epsilon')$ denote next-period liabilities, contingent on the realization of the aggregate shock ϵ' , if tomorrow is not a terminal date, and let $b'_L(\epsilon')$ denote the same, in the event of termination. Then, the entrepreneur faces the constraint

$$b = d + \beta_C ((1 - \gamma) \mathbb{E}[b'(\epsilon')] + \gamma \mathbb{E}[b'_L(\epsilon)]), \quad (2)$$

where the expectation is taken with respect to ϵ' .

The entrepreneur has to ensure that his future promised repayments are credible. Recall that, if the entrepreneur defaults, his liabilities are set to zero and he has access to a fraction $(1 - \theta)$ of the capital. Therefore, if tomorrow is a continuation date, his promised repayments $b'(\epsilon')$ have to satisfy the no-default condition

$$V(k', b'(\epsilon'), X') \geq V((1 - \theta) k', 0, X') \quad (3)$$

for all ϵ' . Throughout this section, X' stands for $H(X, \epsilon')$. If tomorrow is the final period, the entrepreneur can either liquidate his firm, getting $R(X') k'$, and repay his liabilities, or default and get $(1 - \theta) R(X') k'$. Therefore, the no-default condition in the final period takes the form

$$R(X') k' - b'_L(\epsilon') \geq (1 - \theta) R(X') k', \quad (4)$$

which again needs to hold for all ϵ' .

We are now ready to write the Bellman equation for the entrepreneur:

$$\begin{aligned} V(k, b, X) &= \max_{c^E, k', b'(\cdot), b'_L(\cdot)} c^E + \beta_E (1 - \gamma) \mathbb{E} [V(k', b'(\epsilon'), X')] + \\ &\quad + \beta_E \gamma \mathbb{E} [R(X') k' - b'_L(\epsilon')] \end{aligned} \quad (P)$$

s.t. (1), (2), (3) and (4).

Notice that, except for constraint (3), all constraints are linear in the individual states k and b , and in the choice variables $c^E, k', b'(\cdot)$ and $b'_L(\cdot)$. Let us make the conjecture that

the value function is linear and takes the form

$$V(k, b, X) = \phi(X)(R(X)k - b), \quad (5)$$

for some positive, state-contingent function $\phi(X)$. Then, the no-default condition (3) becomes linear as well, and can be rewritten as

$$b'(\epsilon') \leq \theta R(X')k'. \quad (3')$$

This is a form of “collateral constraint,” which implies that an entrepreneur can only pledge a fraction θ of the future gross returns $R(X')k$.⁹ The crucial difference with similar constraints in the literature (e.g., in Kiyotaki and Moore (1997)), is the fact that we allow for fully state-contingent securities.

Before stating Proposition 3, we impose some restrictions on the equilibrium prices $w(\cdot)$ and $q^o(\cdot)$ and on the law of motion H . These conditions ensure that the entrepreneur’s problem is well defined and deliver a simple optimal contract where the collateral constraint (3’) is always binding. In subsection 2.4 we will verify that these conditions are met in equilibrium.

Suppose the law of motion H admits an ergodic distribution for the aggregate state X , with support \mathbf{X} . Assume that equilibrium prices are such that the following inequalities hold for each $X \in \mathbf{X}$:

$$\beta_E \mathbb{E}[R(X')] > q^m(X), \quad (6)$$

$$\theta \beta_C \mathbb{E}[R(X')] < q^m(X), \quad (7)$$

and

$$\frac{(1-\gamma)(1-\theta)\mathbb{E}[R(X')]}{q^m(X) - \theta\beta_C\mathbb{E}[R(X')]} < 1. \quad (8)$$

Condition (6) implies that the expected rate of return on capital $\mathbb{E}[R(X')]/q^m(X)$ is greater than the inverse discount factor of the entrepreneur, so a continuing entrepreneur prefers investment to consumption. Condition (7) implies that “pledgeable” returns are insufficient to finance the purchase of one unit of capital, i.e., investment cannot be fully financed with outside funds. This condition ensures that investment is finite. Finally, condition (8) ensures that the entrepreneur’s utility is bounded.

Before introducing one last condition, we need to define a function ϕ , which summarizes information about current and future prices.

Lemma 2 *When conditions (6)-(8) hold, there exists a unique function $\phi : \mathbf{X} \rightarrow [1, \infty)$*

⁹Constraint (4) immediately gives an analogous inequality for b'_L .

that satisfies the following recursive definition

$$\phi(X) = \frac{\beta_E (1 - \theta) \mathbb{E}[(\gamma + (1 - \gamma) \phi(X')) R(X')]}{q^m(X) - \theta \beta_C \mathbb{E}[R(X')]} \quad (9)$$

This function satisfies $\phi(X) > 1$ for all $X \in \mathbf{X}$.

A further condition on equilibrium prices is then:

$$\phi(X) > \frac{\beta_E}{\beta_C} \phi(X') \quad (10)$$

for all $X \in \mathbf{X}$ and all $X' = H(X, \epsilon')$. Condition (10) ensures that entrepreneurs never delay investment. Namely, it implies that they always prefer to invest in physical capital today rather than buying a state-contingent security that pays in some future state.

The function ϕ defined in Lemma 2 will play a central role in the rest of the analysis. The next proposition shows that substituting $\phi(X)$ on the right-hand side of (5), gives us the value function for the entrepreneur (justifying our slight abuse of notation). Define the *net worth* of the entrepreneur

$$n(k, b, X) \equiv R(X)k - b,$$

which represents the difference between the liquidation value of the firm and the value of its liabilities. Equation (5) implies that expected utility is a linear function of net worth and $\phi(X)$ represents the *marginal value of entrepreneurial net worth*. We will go back to its interpretation in subsection 2.5.

Proposition 3 *Suppose the aggregate law of motion H and the equilibrium prices $w(\cdot)$ and $q^\circ(\cdot)$ are such that (6)-(8) and (10) hold, where $\phi(\cdot)$ is defined as in Lemma 2. Then, the value function $V(k, b, X)$ takes the form (5) and the entrepreneur's optimal policy is*

$$c^E = 0, \quad (11)$$

$$k' = \frac{R(X)k - b}{q^m(X) - \theta \beta_C \mathbb{E}[R(X')]}, \quad (12)$$

$$b'(\epsilon') = b'_L(\epsilon') = \theta R(X')k'. \quad (12)$$

The entrepreneur's problem can be analyzed under weaker versions of conditions (6)-(8) and (10). However, as we shall see in a moment, these conditions are appropriate for studying small stochastic fluctuations around a steady state where the financial constraint is binding.

2.4 Aggregation

Having characterized optimal individual behavior, we now aggregate and impose market clearing on the labor market and on the used capital market. To help the reading of the dynamics, we now revert to using time subscripts.

Each period, a fraction γ of entrepreneurs begins life with zero capital and labor income $w_t l_E$. Their net worth is simply equal to their labor income. Moreover, a fraction $(1 - \gamma)$ of continuing entrepreneurs has net worth equal to $n_t = R_t k_t - b_t$. The aggregate net worth of the entrepreneurial sector, excluding entrepreneurs in the last period of activity, is then given by

$$N_t = (1 - \gamma) (R_t K_t - B_t) + \gamma w_t l_E.$$

Using the optimal individual rules (11) and (12), we get the following dynamics for the aggregate states K_t and B_t

$$K_{t+1} = \frac{(1 - \gamma) (R_t K_t - B_t) + \gamma w_t l_E}{q_t^o - \theta \beta_C \mathbb{E}_t [R_{t+1}]}, \quad (13)$$

$$B_{t+1} = \beta_C \theta R_{t+1} K_{t+1}. \quad (14)$$

Finally, the following conditions ensure that the prices w_t and q_t^o are consistent with market clearing in the labor market and in the used capital market

$$w_t = A_t \frac{\partial F(K_t, 1)}{\partial L_t}, \quad (15)$$

$$q_t^o = -\frac{\partial G(K_{t+1}, K_t)}{\partial K_t}. \quad (16)$$

To clarify the role of condition (16), notice that all continuing entrepreneurs choose the same ratio k_t^o/k_{t+1} , and this ratio must satisfy the first-order condition $q_t^o + \partial G(k_{t+1}, k_t^o) / \partial k_t^o = 0$. Market clearing on the used capital market requires that continuing entrepreneurs acquire all the existing used capital stock, so K_t/K_{t+1} is equal to k_t^o/k_{t+1} . This gives us condition (16).

Summing up, we have found a recursive equilibrium if the laws of motion \mathcal{K} and \mathcal{B} and the pricing rules for w_t and q_t^o satisfy (13) to (16), and if they are such that conditions (6)-(10) are satisfied. The next proposition shows that an equilibrium with these properties exists under some parametric assumptions. Let the production function and the adjustment cost function be:

$$A_t F(k_t, l_t) = A_t k_t^\alpha l_t^{1-\alpha}, \quad (17)$$

$$G(k_{t+1}, k_t) = k_{t+1} - (1 - \delta) k_t + \frac{\xi}{2} \frac{(k_{t+1} - k_t)^2}{k_t}. \quad (18)$$

To construct a recursive equilibrium, we consider a deterministic version of the same economy (i.e., an economy where A_t is constant and equal to 1), and use the deterministic steady

state as a reference point. Let $[\underline{A}, \overline{A}]$ be the support of A_t in the stochastic economy.

Proposition 4 *Consider an economy with Cobb-Douglas technology and quadratic adjustment costs. Consider the steady-state equilibrium of the corresponding deterministic economy and suppose that it satisfies the following two properties: (i) the financial constraint is binding at the steady state ($\beta R^{SS} > 1$), (ii) the steady state is locally saddle-path stable. Then there is a scalar $\Delta > 0$ such that, if $\overline{A} - \underline{A} < \Delta$, there exists a recursive competitive equilibrium of the stochastic economy with aggregate dynamics described by (13)-(14).*

In the appendix we spell out conditions on the economy's underlying parameters which ensure that the deterministic steady state satisfies conditions (i) and (ii) in the hypothesis of this proposition.¹⁰ These parametric restrictions are satisfied in all the calibrations considered below.

Finally, as a useful benchmark, let us briefly characterize the frictionless equilibrium which arises when $\theta = 1$. In the frictionless benchmark, equilibrium dynamics are fully characterized by the condition

$$q_t^m = \beta_C \mathbb{E}_t [R_{t+1}]. \quad (19)$$

The definitions of q_t^m and R_t are the same as those given in the constrained economy, and so are the equilibrium conditions (15) and (16) for w_t and q_t^o . Given that $\beta_E < \beta_C$ entrepreneurs consume their wealth $w_t l_E$ in the first period of their life and consume zero in all future periods. Investment is entirely financed by consumers, which explains why the consumers' discount factor appears in the equilibrium condition (19).

2.5 Average q and marginal q

Having characterized equilibrium dynamics, we can now derive the appropriate expressions for average q and for marginal q . Marginal q is immediately derived from the entrepreneur's problem as the shadow value of new capital, q_t^m . The definition of q_t^m in Lemma 1 and the equilibrium condition (16) can be used to obtain

$$q_t^m = \frac{\partial G(K_{t+1}/K_t, 1)}{\partial K_{t+1}}.$$

This is the standard result in economies with convex adjustment costs: there is a one-to-one relation between the investment rate and the shadow price of new capital.

To derive average q , we first need to obtain the financial value of a representative firm, that is, the sum of the value of all the claims on the firm's future earnings, held by insiders (entrepreneurs) and outsiders (consumers). For firms in the last period of activity this value is zero. For continuing firms, this gives us the expression

$$p_t \equiv V(k_t, b_t, X_t) + b_t - d_t. \quad (20)$$

¹⁰See conditions (33) and (35) in the proof of Proposition 4.

We subtract the current payments to outsiders, d_t , to obtain the end-of-period value of the firm. Recall that continuing entrepreneurs receive zero payments in the optimal contract (except in the final date), so there is no need to subtract current payments to insiders.

Dividing the financial value of the firm by the total capital invested we obtain average q

$$q_t \equiv \frac{p_t}{k_{t+1}}.$$

Notice we divide by k_{t+1} because we are evaluating q_t at the end of the period, after the capital k_{t+1} has been installed. This is consistent with the fact that the value of the firm, p_t , is also evaluated at the end of the period. In the recursive equilibrium described above, q_t is the same for all continuing firms. For liquidating firms both p_t and k_{t+1} are zero, so q_t is not defined for those firms.

The next proposition shows that the financial constraint introduces a wedge between marginal q and average q , and that the wedge is determined by ϕ_t , the marginal value of entrepreneurial wealth.

Proposition 5 *In the recursive equilibrium described in Proposition 4, average q is the same for all continuing firms and is greater than marginal q , $q_t > q_t^m$. Everything else equal, the ratio q_t/q_t^m is increasing in ϕ_t .*

Proof. Substituting the value function in the value of the firm (20), and rearranging gives

$$p_t = (\phi_t - 1)(R_t k_t - b_t) + R_t k_t - d_t. \quad (21)$$

Using the entrepreneur's budget constraint, constant returns to scale for G , and the equilibrium properties of q_t^o and q_t^m , gives

$$\begin{aligned} R_t k_t - d_t &= G(k_{t+1}, k_t^o) + q_t^o k_t^o = \\ &= \frac{\partial G(k_{t+1}, k_t^o)}{\partial k_{t+1}} k_{t+1} + \frac{\partial G(k_{t+1}, k_t^o)}{\partial k_t} k_t^o + q_t^o k_t^o = \\ &= q_t^m k_{t+1}. \end{aligned}$$

Substituting in (21) and rearranging gives

$$q_t = (\phi_t - 1) \frac{R_t k_t - b_t}{k_{t+1}} + q_t^m. \quad (22)$$

Notice that (11) implies that $(R_t k_t - b_t)/k_{t+1}$ is equal across continuing firms. Given that $\phi_t > 1$ and $b_t \leq \theta R_t k_t < R_t k_t$ the stated results follow from this expression. ■

Notice that in the frictionless benchmark investment is fully financed by consumers and we have $b_t = R_t k_t$, which immediately implies $q_t = q_t^m$. In this case, the model boils down to the Hayashi (1982) model: average q is identical to marginal q and is a sufficient statistic for investment.

It is useful to provide some explanation for the wedge between average q and marginal q in the constrained economy. First, notice that this wedge is not due to the difference in the discount factors of entrepreneurs and consumers. In fact, if we evaluated the expected present value of the entrepreneur's payoffs $\{c_{t+j}^E\}$ using the discount factor β_C instead of β_E , we would get a quantity greater than $V(k_t, b_t, X_t)$ and the measured wedge would be larger.¹¹ The fundamental reason why the wedge is positive is that $\phi_t > 1$, the marginal value of entrepreneurial wealth is larger than one. If ϕ_t was equal to 1, then the first term on the right-hand side of (22) would be zero and the wedge would disappear.

To clarify the mechanism, consider an entrepreneur who begins life with one dollar of wealth. Suppose he uses this wealth to start a firm financed only with inside funds and consumes the receipts at date $t + 1$. The (shadow) price of a unit of capital is q_t^m , so the entrepreneur can install $1/q_t^m$ units of capital. In period $t + 1$ he receives and consumes R_{t+1}/q_t^m . The value of the firm for the entrepreneur is then $\beta_E \mathbb{E}_t [R_{t+1}] / q_t^m$ which is greater than one, by condition (6). In short, the value of a unit of installed capital is larger inside the firm than outside the firm, and this explains why q theory does not hold. This discrepancy does not open an arbitrage opportunity, because the agents that can take advantage of this opportunity (the entrepreneurs) are against a financial constraint. This thought experiment captures the basic intuition behind Proposition 5.

To go one step further, notice that the entrepreneur can do better than following the strategy described above. In particular, he can use borrowed funds on top of his own funds, and he can re-invest the revenues made at $t + 1$, rather than consume. The ability of borrowing allows the entrepreneur to earn an expected *leveraged* return, between t and $t + 1$, equal to¹²

$$(1 - \theta) \mathbb{E}_t [R_{t+1}] / (q_t^m - \theta \beta_C \mathbb{E}_t [R_{t+1}]) > \mathbb{E}_t [R_{t+1}] / q_t^m.$$

Iterating expression (9) forward shows that ϕ_t is a geometric cumulate of future leveraged returns discounted at the rate β_E , taking into account the fact that, as long as the entrepreneur remains active, he can reinvest the returns made in his firm. Therefore, when borrowing and reinvestment are taken into account, one dollar of wealth allows the entrepreneur to obtain a value of $\phi_t > \beta_E \mathbb{E}_t [R_{t+1}] / q_t^m > 1$. At the same time, the entrepreneur is receiving $q_t^m k_{t+1} - 1$ from outside investors (recall that he only has 1 dollar of internal funds). Therefore, the value of the claims issued to outsiders must equal $q_t^m k_{t+1} - 1$. In conclusion, an entrepreneur with one dollar to invest can start a firm valued at $\phi_t + q_t^m k_{t+1} - 1$, which is larger than the value of invested capital, $q_t^m k_{t+1}$, given that $\phi_t > 1$.

¹¹For the quantitative results presented in Section 3, we also experimented with this alternative definition of q (discounting the entrepreneur's claims at the rate β_C instead of β_E), with minimal effects on the results.

¹²Notice that, from (11), $1 / (q_t^m - \theta \beta_C \mathbb{E}_t [R_{t+1}])$ is the capital stock k_{t+1} which can be invested by an entrepreneur with one dollar of wealth. In $t + 1$ the entrepreneur has to repay $\theta R_{t+1} k_{t+1}$ and can keep $(1 - \theta) R_{t+1} k_{t+1}$. To prove the inequality, rearrange it and simplify to obtain

$$\theta \beta_C (\mathbb{E}_t [R_{t+1}])^2 > \theta q_t^m \mathbb{E}_t [R_{t+1}].$$

The inequality follows from (a) and $\beta_C > \beta_E$.

3 Quantitative Implications

In this section, we examine the quantitative implications of the model looking at the joint behavior of investment, Tobin’s q , and cash flow in a simulated economy. First, we give a basic quantitative characterization of the economy’s response to a productivity shock. Second, we ask whether the wedge between marginal q and average q in our model helps to explain the empirical failure of q theory in investment regressions.

3.1 Baseline calibration

The production function is Cobb-Douglas and adjustment costs are quadratic, as specified in (17) and (18). The productivity process is given by $A_t = e^{a_t}$, where a_t follows the autoregressive process

$$a_t = \rho a_{t-1} + \epsilon_t,$$

with ϵ_t a Gaussian, i.i.d. shock.¹³

β_C	0.97	β_E	0.96
α	0.33	δ	0.05
ξ	8.5	ρ	0.75
θ	0.3	γ	0.06
l_E	0.2		

Table 1. Baseline calibration.

The baseline parameters for our calibration are reported in Table 1. The time period is a year, so we set β_C to give an interest rate of 3%. For the discount factor of the entrepreneurs, we choose a value smaller but close to that of the consumers. The values for α and δ are standard. The values of ξ and ρ are chosen to match basic features of firm-level data on cash flow and investment. In particular, we consider the following statistics, obtained from the Compustat dataset.¹⁴

$r(CFK)$	$\sigma(IK)$	$\sigma(CFK)$
0.51	0.061	0.128

where CFK denotes cash flow per unit of capital invested, IK denotes the investment rate, $r(\cdot)$ denotes the (yearly) coefficient of serial correlation, and $\sigma(\cdot)$ the standard deviation.

¹³The theoretical analysis can be extended to the case where ϵ_t is a continuous variable. To ensure that A_t is bounded, we set $A_t = \underline{A}$ whenever $e^{a_t} < \underline{A}$ and $A_t = \bar{A}$ whenever $e^{a_t} > \bar{A}$. As long as σ_ϵ^2 is small the bounds \underline{A} and \bar{A} are immaterial for the results.

¹⁴We use the same data from Compustat as Gilchrist and Himmelberg (1995). The sample consists of 428 U.S. stock market listed firms from 1978 to 1989. We use the code of João Ejarque to calculate firm-specific statistics separately for each variable. The moments reported in this paper are the means across all firms. Any ratio used (e.g. $\sigma(IK)/\sigma(CFK)$) is a ratio of such means.

tion. We calibrate ρ so that our simulated series replicate the autocorrelation of cash flow $r(CFK) = 0.51$. In our baseline calibration this gives us $\rho = 0.75$. We set ξ to match the ratio between cash flow volatility and investment volatility, $\sigma(IK)/\sigma(CFK) = 0.48$. Given all the other parameters, this gives us $\xi = 8.5$.

Finally, the parameters θ , γ , and l_E are chosen as follows. Fazzari, Hubbard and Petersen (1988) report that 30% of manufacturing investment is financed externally. Based on this, we choose $\theta = 0.3$. The parameters γ and l_E are chosen to obtain an outside finance premium of 2%, as in Bernanke, Gertler and Gilchrist (2000). We experimented with different values of γ and l_E and found that, as long as the finance premium remains at 2%, the specific choice of these two parameters has minimal effects on our results.

3.2 Impulse responses

In the model, the net investment rate of the representative firm is

$$IK_t \equiv \frac{k_{t+1} - (1 - \delta)k_t}{k_t},$$

and the ratio of cash flow to the firm's capital stock is

$$CFK_t \equiv \frac{A_t F(k_t, l_t) - w_t l_t}{k_t}.$$

Figure 1 plots the responses of IK_t , q_t , and CFK_t , following a positive technology shock. All variables are expressed in terms of deviations from their steady-state values.

All three variables in Figure 1 increase on impact, as in the standard model without financial frictions. However, the dynamics of average q are now jointly determined by marginal q and by the wedge q_t/q_t^m . Marginal q moves one for one with investment. Average q initially rises with investment, but at some point (3 periods after the shock) it falls below its steady-state value, while investment continues to be above the steady state for several more periods (up to period 6 periods after the shock). As marginal q is reverting towards its steady state the wedge remains large, thus pushing average q below the steady state. The slow-moving dynamics of the wedge are responsible for breaking the synchronicity between average q and investment.

In Proposition 5, we argued that the ratio of average q to marginal q is positively related to ϕ_t , the marginal value of entrepreneurial net worth. The top panel of Figure 2 plots the response of ϕ_t to the same technology shock, showing that ϕ_t decreases on impact following the shock, and then slowly reverts to its steady-state value. The slow adjustment in the wedge is closely related to the slow adjustment of ϕ_t .

To understand the response of ϕ_t , recall from the discussion in subsection 2.5 that the dynamics of ϕ_t are closely related to those of the rate of return $\mathbb{E}_t[R_{t+1}]/q_t^m$, since ϕ_t is a forward-looking measure which cumulates the discounted returns on entrepreneurial

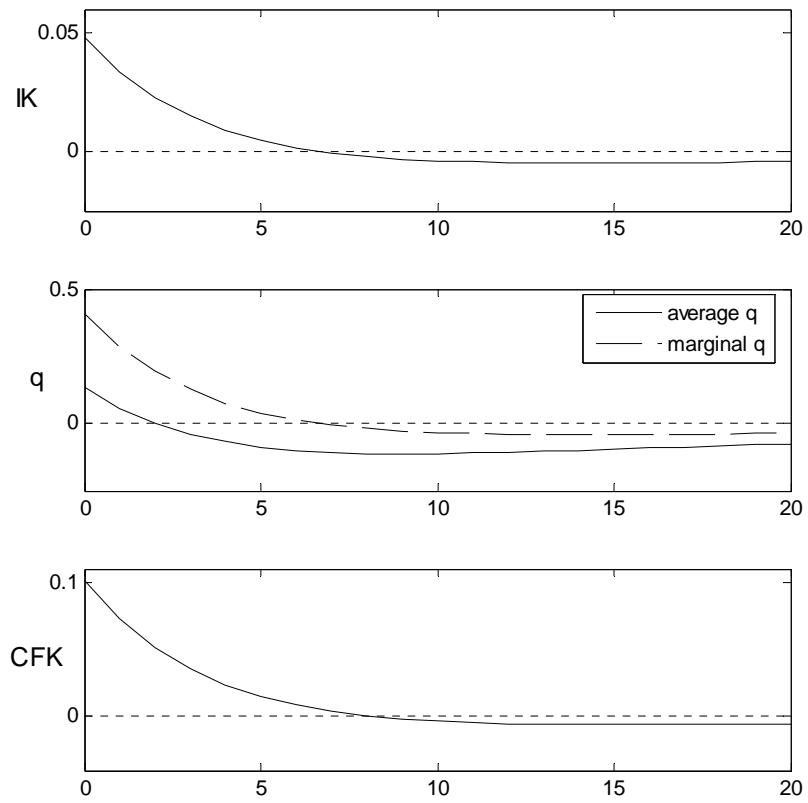


Figure 1: Responses of investment, q , and cash flow to a technology shock.

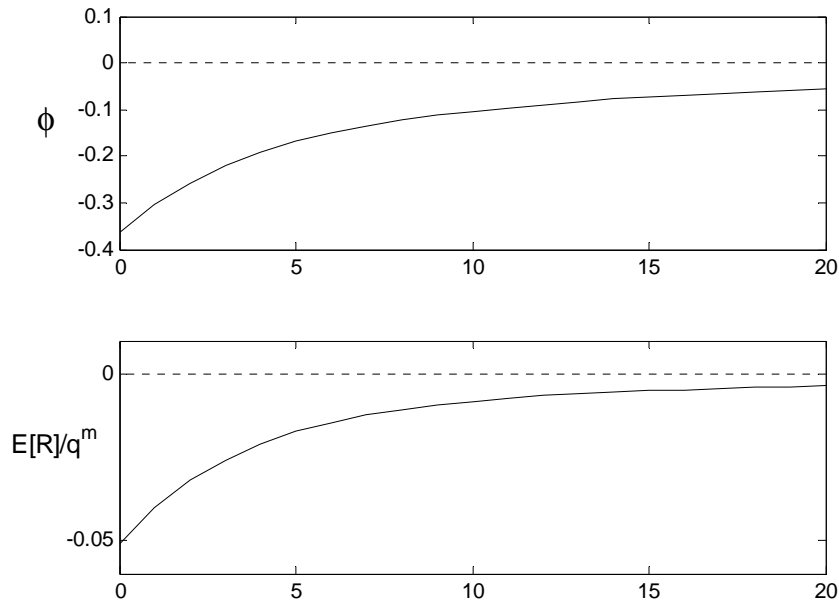


Figure 2: Responses of ϕ and expected returns to a technology shock.

investment in all future periods. The dynamics of ϕ_t reflect the fact that the rate of return on entrepreneurial investment drops following a positive technology shock, as shown in the bottom panel of Figure 2.¹⁵ Two opposite forces are at work here. First, due to the persistent nature of the shock, future productivity increases and this raises expected returns per unit of capital, R_{t+1} . This tends to increase the marginal value of entrepreneurial wealth. At the same time, entrepreneurs' net worth increases because of the current increase in cash flow. This leads to an increase in K_{t+1} , which reduces R_{t+1} , due to decreasing returns to capital, and increases q_t^m , due to adjustment costs. These effects tend to reduce the marginal value of entrepreneurial wealth. In the case considered, the second channel dominates and the net effect is a reduction in $\mathbb{E}_t[R_{t+1}]/q_t^m$ and in ϕ_t . As we will see in subsection 3.5, this result depends of the type of shock considered, and can be reversed if we consider shocks with greater persistence. For now, what matters is that the dynamic response of ϕ_t breaks the one-to-one correspondence between IK_t and q_t .

3.3 Investment regressions

We now turn to investment regressions, and ask whether our model can replicate the coefficients on q and cash flow observed in the data. To do so, we generate simulated time series

¹⁵Since the market rate of return is constant and equal to $1/\beta_C$, this also implies that the “outside finance premium” $\mathbb{E}_t[R_{t+1}]/q_t^m - 1/\beta_C$ decreases following a positive shock.

from our calibrated model and run the standard investment regression

$$IK_t = a_0 + a_1q_t + a_2CFK_t + e_t. \quad (23)$$

The regression coefficients for the simulated model are presented in the first row of Table 2. As reference points, we report the coefficients that arise in the model without financial frictions ($\theta = 1$) and the empirical coefficients obtained by Gilchrist and Himmelberg (1995). The latter are representative of the orders of magnitude obtained in empirical studies. Absent financial frictions, q is a sufficient statistic for investment, so the model gives a coefficient on cash flow equal to zero. In this case, the coefficient on q is equal to $1/\xi$, which, given the calibration above is equal to 0.118, a value substantially higher than those obtained in empirical regressions. Adding financial frictions helps both to obtain a positive coefficient on cash flow and a smaller coefficient on q . The impulse response functions reported in Figure 1 help us to understand why. Financial frictions weaken the relation between i_t and q_t , while investment and cash flow remain closely related, due to the effect of cash flow on entrepreneurial net worth.

	a_1	a_2
Model with financial friction	0.018	0.444
Frictionless model	0.118	0.000
Gilchrist and Himmelberg (1995)	0.033 (0.016)	0.242 (0.038)

Table 2. Investment regressions.

Third line: Standard errors in parenthesis.

Notice that under the simple AR1 structure for productivity used here, a sizeable correlation between q and investment is still present. Running a simple univariate regression of investment on q gives a coefficient of 0.13, not too far from the frictionless coefficient, and an R^2 of 0.5. This is not surprising, given that only one shock is present. However, once cash flow is added to the independent variables, the explanatory power of q falls dramatically. To see this, notice that the R^2 of the bivariate regression is virtually 1, while the R^2 of a univariate regression of investment on cash flow alone is 0.995. So the additional explanatory power of q is less than 1 percent of investment volatility.

The values of R^2 just reported are clearly unrealistic and are a product of the simple one-shock structure used. Furthermore, idiosyncratic uncertainty and measurement error are absent from the exercise. For these reasons, we do not attempt to exactly replicate the empirical coefficients for q and cash flow.¹⁶ Instead, our point here is that a reasonable calibration of the model can help generate realistic coefficients for both q and cash flow, by introducing a time-varying wedge between marginal q and average q . An extension of the

¹⁶By changing the model parameters, in particular increasing θ and ξ , it is possible to match exactly the coefficients in Gilchrist and Himmelberg (1995).

model that allows for idiosyncratic uncertainty is discussed below.

3.4 Sensitivity

To verify the robustness of our result, we experiment with different parameter configurations, in a neighborhood of the parameters introduced above. Table 3 shows the coefficients of the investment regression for a sample of these alternative specifications. Note that our basic result holds under a large set of possible parameterizations. Moreover, a number of interesting comparative statics patterns emerge.

	a_1	a_2
Baseline	0.018	0.44
$\theta = 0.2$	0.012	0.50
$\theta = 0.4$	0.025	0.39
$\alpha = 0.2$	0.022	0.45
$\alpha = 0.4$	0.017	0.44
$\xi = 4$	0.022	0.67
$\xi = 12$	0.015	0.35
$l_E = 0.1$	0.017	0.44
$l_E = 0.3$	0.019	0.45
$\rho = 0.6$	0.023	0.36
$\rho = 0.9$	0.011	0.57

Table 3. Sensitivity analysis.

First, notice that increasing θ brings the economy closer to the frictionless benchmark and reduces the wedge between marginal q and average q . This accounts for the increase in the coefficient on q and the decrease of the coefficient on cash flow when we increase θ . However, this comparative static result does not apply to all parameter changes that bring the economy closer to the frictionless benchmark. In particular, notice that when we increase l_E (which determines the initial wealth of the entrepreneurs) both the coefficient on q and the coefficient on cash flow increase.¹⁷ This is consistent with the general point raised by Kaplan and Zingales (1997), who note that the coefficient on cash flow in investment regressions is not necessarily a good measure of how tight the financial constraint is.

Increasing ξ reduces the response of investment to the productivity shock and decreases the coefficients of both q and cash flow. Finally, an increase in the persistence of the technology shock, ρ , tends to lower the coefficient on q and to increase the coefficient on cash flow. The effect of changing ρ is analyzed in detail in the following subsection.

¹⁷A similar result emerges if we decrease γ .

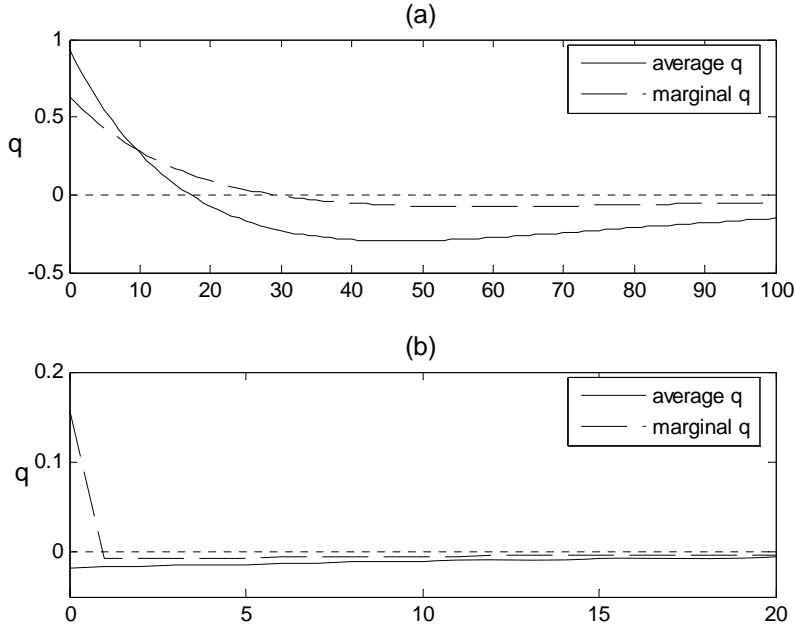


Figure 3: Responses of q and q^m to a technology shock for different degrees of persistence. Panel (a): $\rho = 0.98$. Panel (b): $\rho = 0$.

3.5 Current and future changes in productivity

To further clarify what determines the wedge between marginal and average q , it is useful to compare the effect of shocks with different persistence. Figure 3 plots the impulse-response functions of average q and marginal q for two different values of the autocorrelation coefficient, ρ . They can be compared to the middle panel in Figure 1.

In panel (a) of Figure 3 we plot the effect of a very persistent shock ($\rho = 0.98$). In this case, the effect of the shock on future returns dominates the effect on current cash flow. Entrepreneurial investment becomes very profitable while entrepreneurs' internal funds are only catching up gradually. The wedge increases in the short-run, reflecting the fact that the financial constraint is initially tighter. In panel (b) we plot the effect of a temporary shock ($\rho = 0$). This shock has the opposite effect on the wedge on impact: internal funds are higher, while future total factor productivity is unchanged. As investment increases, the equilibrium rate of return $\mathbb{E}_t [R_{t+1}] / q_t^m$ falls due to decreasing returns to capital and convex adjustment costs. The wedge falls, and this effect is so strong that average q and marginal q move in opposite directions. Marginal q increases, due to the increase in investment, while average q falls reflecting the lowered expected profitability of entrepreneurial investment.

The two plots in Figure 3 show that the wedge between marginal and average q captures the tension between the future profitability of investment and the current availability of funds to the entrepreneur. They also suggest that the observed volatility of q depends on

the types of shocks hitting the economy. In Table 4 we report the ratio of the volatility of q to the volatility of the investment rate, $\sigma(q)/\sigma(IK)$, for different values of ρ . For comparison, the value of the ratio $\sigma(q)/\sigma(IK)$ for Compustat firms is equal to 27.¹⁸

In the frictionless benchmark, the ratio between asset price volatility and investment volatility is equal to ξ , which we are keeping constant at 8.5. For values of ρ lower than 0.89 the presence of the financial friction tends to dampen asset price volatility. However, for higher values of ρ , asset price volatility is amplified. For example, when $\rho = 0.98$ the volatility of q doubles compared to frictionless case, although it is still smaller than in the data. Highly persistent shocks to productivity help to obtain more volatile asset prices, by generating variations in the long run expected return on entrepreneurial capital. The role of shocks to future productivity in magnifying asset price volatility has recently been emphasized in Abel and Eberly (2005), in the context of a model with no financial frictions, but with decreasing returns and market power. This exercise suggests that a model with constant returns and financial frictions can lead to similar conclusions. The highly persistent shock considered here is a combination of a change in current productivity and a change in future productivity. The explicit treatment of pure “news shocks,” only affecting future productivity, is left to future work (Walentin (2007)).

ρ	0	0.25	.50	0.75	0.98	0.967
ξ	8.5	8.5	8.5	8.5	8.5	18
$\sigma(q)/\sigma(IK)$	1.9	2.3	3.4	5.5	16.5	27
$\sigma(IK)/\sigma(CFK)$	0.19	0.24	0.33	0.48	0.73	0.48

Table 4. Shock persistence and the volatility of q .

In Table 4 we also report the effects of different values of ρ on the volatility of investment $\sigma(IK)/\sigma(CFK)$. High values of ρ tend to increase the volatility of investment relative to the volatility of cash flow. When we increase ρ we can re-calibrate ξ to keep $\sigma(IK)/\sigma(CFK) = 0.48$ (as in the baseline calibration above) and this leads to a further increase in the volatility of q . In particular, setting $\rho = 0.967$ and $\xi = 18$, allows us to match the empirical values of both $\sigma(IK)/\sigma(CFK)$ and $\sigma(q)/\sigma(IK)$ (see the last column of Table 4). Although the model does well on these dimensions, the required adjustment costs seems very high and this parametrization delivers an excessive degree of serial correlation for cash flow. A relatively easy fix would be to introduce a combination of both temporary shocks and shocks to long-run productivity. This would allow the model to deliver less serial correlation, while at the same time having larger movements in q that are uncorrelated with current investment. Again, this extension is better developed in a model that allows for a richer set of shocks and is left to future work.

¹⁸See footnote 14 for calculation method.

4 Firm-level Heterogeneity

So far, we have focused on an economy where all firms have the same productivity, and only aggregate productivity shocks are present. This, together with the assumption of constant returns to scale, implies that the investment rate, q , and cash flow (normalized by assets) are identical across firms. The advantage of this approach is that it makes it easy to compare our results to the classic Hayashi (1982) model. At the same time, this approach has its limitations, given that the evidence on the relation between q and investment is largely based on panel data. Therefore, it is useful to consider variations of the model that allow for cross-sectional heterogeneity.

An immediate extension is to allow for multiple sectors. If we assume that labor and capital are immobile across sectors, which may be a reasonable approximation in the short run, then w and q^o are sector-specific prices and each sector's dynamics are analogous to the aggregate dynamics studied above. Therefore, under this interpretation, all the results presented so far apply to the multiple sector case. In this section, we pursue an alternative extension, by introducing productivity differences across firms. Let $A_{j,t}$ denote the productivity of firm j . Newborn entrepreneurs receive an initial random draw $A_{j,t}$ from a given distribution Φ . From then on, individual productivity follows the stationary process $A_{j,t} = \Gamma(A_{j,t-1}, \epsilon_{j,t})$ with $\epsilon_{j,t}$ drawn from the discrete p.d.f. $\pi(\epsilon_{j,t})$. To keep matters simple, we abstract from aggregate uncertainty and assume that the realized cross-sectional distribution of the shocks is always identical to the ex-ante distribution for each individual firm. The details of this extension are presented in Appendix B.

Given the absence of aggregate uncertainty, aggregate capital is constant in this economy and so is the wage w and the price of used capital q^o . This also implies that q^m is constant and equal to 1. However, as long as the financial constraint is binding, average q is greater than 1 and is different across firms. The assumption of constant returns to scale still helps to simplify the problem, as it implies that the investment rate, average q , and the cash-flow-to-assets ratio are independent of the individual firm's assets $k_{j,t}$. However, these three variables are now functions of the firm's productivity $A_{j,t}$ and are given by the following three equations,

$$IK_{j,t} = \frac{(1 - \theta) R_{j,t} - 1}{1 - \theta \beta_C \mathbb{E}[R_{j,t+1}|A_{j,t}]} - 1,$$

$$q_{j,t} = \beta_E (1 - \theta) \mathbb{E}[(\gamma + (1 - \gamma) \phi_{j,t+1}) R_{j,t+1}|A_{j,t}] + \theta \beta_C \mathbb{E}[R_{j,t+1}|A_{j,t}],$$

$$CFK_{j,t} = R_{j,t} - q^o,$$

where the return per unit of capital, $R_{j,t}$, and the marginal value of entrepreneurial wealth, $\phi_{j,t}$, are now firm-specific variables.¹⁹

The three expressions above for $IK_{j,t}$, $q_{j,t}$, and $CFK_{j,t}$, emphasize once more the tension

¹⁹Both $R_{j,t}$ and $\phi_{j,t}$ are functions only of $A_{j,t}$, so the distributions of $R_{j,t+1}$ and $\phi_{j,t+1}$ conditional on $A_{j,t}$ can be obtained from the law of motion $A_{j,t+1} = \Gamma(A_{j,t}, \epsilon_{j,t+1})$.

between current and future changes in productivity discussed in subsection 3.5. On the one hand, current returns, captured by $R_{j,t}$, affect positively both the investment rate and cash flow, but have no effect on q , which is a purely forward-looking variable. On the other hand, future returns, captured by $\mathbb{E}[R_{j,t+1}|A_{j,t}]$, affect positively investment and q , but have no effects on current cash flows.

To study the implications of the model for investment regressions, we construct simulated time-series from the model described and run the investment regression (23). In Table 5 we report the regression coefficients obtained from the simulated series, using the same parameters as in Section 3.

	a_1	a_2
Model with financial friction	0.116	1.023

Table 5. Investment regression. Firm-specific shocks.

Once more, financial frictions introduce a strong correlation between cash flow and investment, so that cash flow has a positive coefficient in the regression. Notice that both coefficients a_1 and a_2 are now larger than in the corresponding line of Table 2 and larger than their empirical counterpart. This is not surprising, given that firms now face essentially zero adjustment costs. In this model, adjustment costs are only due to aggregate changes in the capital stock, and with no aggregate uncertainty such changes are absent.²⁰ Another implication of the absence of adjustment costs is that investment is too volatile. The ratio $\sigma(IK)/\sigma(CFK)$ is equal to 1.34 in the simulated series, more than twice as large as in the data.²¹ In our model we have essentially assumed “external adjustment costs,” by allowing firms to trade homogeneous capital on the used capital market. A fully developed model with firm-specific shocks clearly calls for the introduction of “internal adjustment costs,” both to reduce investment volatility at the firm level and to obtain more realistic coefficients in investment regressions. However, with internal adjustment costs we lose analytical tractability, as optimal investment rules are, in general, non-linear.

5 Conclusions

In this paper, we have developed a tractable framework to study the effect of financial frictions on the joint dynamics of investment and of the value of the firm. The model shows that, in the presence of financial frictions, q reflects future quasi-rents that will go to the insider. This introduces a wedge between average and marginal q . The size of this wedge

²⁰The parameter ξ is accordingly irrelevant for this version of the model.

²¹Notice also that the frictionless model is not a very useful benchmark in this case, as it gives very extreme and unrealistic results. Absent financial frictions all the capital stock in the economy would go, each period, to the single firm with the highest expected return on capital, while q would be constant and equal to 1.

is determined by the tension between current and future profitability. A firm with high future productivity and low internal funds today will display a higher q . The reason for this is that the growth of its capital stock is constrained relative to expected productivity, and this raises the future marginal product of capital.

The paper focuses on the implications of the model for the correlation between investment, q and cash flow. In particular, we show that a model with financial frictions can help to replicate the observed low correlation between q and investment, and the fact that cash flow appears with a positive coefficient in standard investment regressions. However, the model has a number of additional testable predictions on the response of investment and asset prices to different types of shocks (shocks with different persistence, shocks affecting current/future productivity), as discussed in Section 3.5. As we noticed, recent models with market power and decreasing returns at the firm level also display rich dynamics following shocks with different temporal patterns. Empirical work documenting the conditional behavior of investment and q following these shocks, would provide an important testing ground for both classes of models.

Throughout the paper, we have maintained Hayashi's (1982) assumption of constant returns to scale both in the production function and in adjustment costs. This has two advantages. First, it greatly simplifies aggregation. Second, it allows us to focus on the "pure" effect of the financial friction on investment regressions. Models with decreasing returns at the firm level can produce deviations from q theory for independent reasons, so it is useful, at this stage, to separate those effects from the effects due to imperfections in financial contracts. At the same time, this choice leaves aside a number of interesting issues, which seem especially relevant when one introduces firm-level heterogeneity, as we did in Section 4.

In the paper we have focused on the case of small stochastic deviations from the steady state. It is possible to extend the model to allow for "large" shocks, opening the door to potentially interesting phenomena. In particular, with large shocks it is possible to have a model where firms hold precautionary reserves, i.e., choose to reduce investment today in order to buy financial securities as insurance against future shocks. This is another area where equilibrium behavior will be very sensitive to the time profile of the shocks hitting the firm.

Finally, in the model we have assumed that consumers are risk-neutral. It turns out that the characterization result in Proposition 4 can be extended to economies with risk-averse consumers. Such an extension, which we leave to future work, would be useful to analyze the model's implications for the aggregate behavior of interest rates and risk-premia.

Appendix

A. Proofs

Proof of Lemma 1

Consider the problem

$$\min_{k^o} G(k', k^o) + q^o k^o. \quad (24)$$

Suppose $k^o = \kappa(q^o)$ is optimal for a given q^o and $k' = 1$. Constant returns to scale imply that, given any k' , $k^o = \kappa(q^o) k'$ is a solution to problem (24) and the optimum is equal to $(G(\kappa(q^o), 1) + q^o) k'$. Therefore, we can set

$$q^m(X) \equiv G(\kappa(q^o(X)), 1) + q^o(X),$$

completing the proof of the first part of the lemma. In a similar way, consider the problem

$$\max_l AF(k, l) - wl + q^o k, \quad (25)$$

and suppose $l = \eta(w, q^o, A)$ is optimal for a given triple w, q^o, A and $k = 1$. Constant returns to scale imply that, given any k , $l = \eta(w, q^o, A) k$ is a solution to (25) and the optimum is $(AF(1, \eta(w, q^o, A)) - w\eta(w, q^o, A) + q^o) k$. Setting

$$R(X) \equiv AF(1, \eta(w(X), q^o(X), A)) - w(X)\eta(w(X), q^o(X), A) + q^o(X)$$

completes the proof.

Proof of Lemma 2

Let \tilde{B} be the space of bounded functions $\phi : \mathbf{X} \rightarrow [1, \infty)$. Define the map $T : \tilde{B} \rightarrow \tilde{B}$ as follows

$$T\phi(X) = \frac{\beta_E(1-\theta)\mathbb{E}[(\gamma + (1-\gamma)\phi(H(X, \epsilon')))]R(H(X, \epsilon'))}{q^m(X) - \theta\beta_C\mathbb{E}[R(H(X, \epsilon'))]}.$$

Let us first check that $T\phi \in \tilde{B}$ if $\phi \in \tilde{B}$, so the map is well defined. Notice that conditions (6)-(7) and $\beta_E < \beta_C$ imply that

$$\frac{(1-\theta)\beta_E\mathbb{E}[R(H(X, \epsilon'))]}{q^m(X) - \theta\beta_C\mathbb{E}[R(H(X, \epsilon'))]} > 1.$$

This implies that for any $\phi \in \tilde{B}$ we have

$$\frac{\beta_E(1-\theta)\mathbb{E}[(\gamma + (1-\gamma)\phi(H(X, \epsilon')))]R(H(X, \epsilon'))}{q^m(X) - \theta\beta_C\mathbb{E}[R(H(X, \epsilon'))]} \geq \frac{\beta_E(1-\theta)\mathbb{E}[R(H(X, \epsilon'))]}{q^m(X) - \theta\beta_C\mathbb{E}[R(H(X, \epsilon'))]} > 1, \quad (26)$$

showing that $T\phi(X) \geq 1$. Assumption (8) implies that

$$\frac{\beta_E(1-\theta)\mathbb{E}[R(H(X, \epsilon'))]}{q^m(X) - \theta\beta_C\mathbb{E}[R(H(X, \epsilon'))]} \leq \frac{1}{1-\gamma},$$

so if $\phi(X) \leq M$ for all $X \in \mathbf{X}$, then $T\phi(X) \leq M/(1-\gamma)$ for all $X \in \mathbf{X}$, completing the argument.

Next, we show that T satisfies Blackwell's sufficient conditions for a contraction. The monotonicity of T is easily established. To check that it satisfies the discounting property notice that if

$\phi' = \phi + a$, then

$$T\phi'(X) - T\phi(X) = \frac{\beta_E(1-\gamma)(1-\theta)\mathbb{E}[R(H(X, \epsilon'))]}{q^m(X) - \theta\beta_C\mathbb{E}[R(H(X, \epsilon'))]}a < \beta_E a,$$

where the inequality follows from assumption (8). Since T is a contraction a unique fixed point exists and (26) immediately shows that $\phi(X) > 1$ for all X .

Proof of Proposition 3

Let ϕ be defined as in Lemma 2. We proceed by guessing and verifying that the value function is $V(k, b, X) = \phi(X)(R(X)k - b)$. In the text, we have shown that, under this conjecture, the no-default condition can be rewritten in the form (3'). Therefore, we can rewrite problem (P) as

$$\begin{aligned} \max_{c^E, k', b'(\cdot), b'_L(\cdot)} & c^E + \beta_E(1-\gamma) \sum_{\epsilon'} \pi(\epsilon') [\phi(H(X, \epsilon'))(R(H(X, \epsilon'))k' - b'(\epsilon'))] + \\ & + \beta_E\gamma \sum_{\epsilon'} \pi(\epsilon') [R(H(X, \epsilon'))k' - b'_L(\epsilon')] \end{aligned}$$

$$\begin{aligned} \text{s.t.} \quad & c^E + q^m(X)k' \leq R(X)k - d, \quad (\lambda) \\ & b = d + \beta_C \left((1-\gamma) \sum_{\epsilon'} \pi(\epsilon') b'(\epsilon') + \gamma \sum_{\epsilon'} \pi(\epsilon') b'_L(\epsilon') \right), \quad (\mu) \\ & b'(\epsilon') \leq \theta R(H(X, \epsilon'))k' \text{ for all } \epsilon', \quad (\nu(\epsilon')\pi(\epsilon')) \\ & b'_L(\epsilon') \leq \theta R(H(X, \epsilon'))k' \text{ for all } \epsilon', \quad (\nu_L(\epsilon')\pi(\epsilon')) \\ & c^E \geq 0, \quad (\tau_c) \\ & k' \geq 0, \quad (\tau_k) \end{aligned}$$

where, in parenthesis, we report the Lagrange multiplier associated to each constraint. The multipliers of the no-default constraints are normalized by the probabilities $\pi(\epsilon')$. The first-order conditions for this problem are

$$\begin{aligned} 1 - \lambda + \tau_c &= 0, \\ \beta_E\mathbb{E}[(\gamma + (1-\gamma)\phi')R'] - \lambda q^m(X) + \theta\mathbb{E}[(\nu + \nu_L)R'] + \tau_k &= 0, \\ -\beta_E(1-\gamma)\phi'\pi(\epsilon') + \lambda\beta_C(1-\gamma)\pi(\epsilon') - \nu(\epsilon')\pi(\epsilon') &= 0, \\ -\beta_E\gamma\pi(\epsilon') + \lambda\beta_C\gamma\pi(\epsilon') - \nu_L(\epsilon')\pi(\epsilon') &= 0, \end{aligned}$$

where R' and ϕ' are shorthand for $R(H(X, \epsilon'))$ and $\phi(H(X, \epsilon'))$. We want to show that the values for c^E, k', b' and b'_L in the statement of the proposition are optimal. It is immediate to check that they satisfy the problem's constraints. To show that they are optimal we need to show that $\tau_c = \lambda - 1 > 0$, $\tau_k = 0$, and $\nu(\epsilon'), \nu_L(\epsilon') > 0$ for all ϵ' . Setting $\tau_k = 0$ the second first-order condition gives us

$$\lambda = \frac{(1-\theta)\beta_E\mathbb{E}[(\gamma + (1-\gamma)\phi')R']}{q^m(X) - \theta\beta_C\mathbb{E}[R']}$$

which, by construction, is equal to $\phi(X)$. Then we have

$$\tau_c = \phi(X) - 1 > 0,$$

which follows from Lemma 2,

$$\nu(\epsilon') = (1 - \gamma)(\beta_C \phi(X) - \beta_E \phi(H(X, \epsilon'))) > 0,$$

which follows from condition (10), and

$$\nu_L(\epsilon') = (1 - \gamma)(\beta_C \phi(X) - \beta_E) > 0,$$

which follows from $\phi(X) > 1$ and $\beta_C > \beta_E$. Substituting the optimal values in the objective function we obtain $\phi(X)(R(X)k - b)$ confirming our initial guess.

Proof of Proposition 4

The proof is split in two steps. In the first step, we construct the steady-state equilibrium of the deterministic economy with a binding financial constraint, in the second, we construct an equilibrium of the stochastic economy. First, we derive a useful preliminary result. Applying the envelope theorem to problems (24) and (25) (see the proof of Lemma 1), using the fact that, in equilibrium, the ratio k^o/k' is equal to K_t/K_{t+1} , and the ratio l/k is equal to $1/K_t$, and using condition (16), we obtain the following expressions for q_t^m and R_t :

$$q_t^m = \frac{\partial G(K_{t+1}, K_t)}{\partial K_{t+1}}, \quad (27)$$

$$R_t = A_t \frac{\partial F(K_t, 1)}{\partial K_t} - \frac{\partial G(K_{t+1}, K_t)}{\partial K_t}. \quad (28)$$

Step 1. (Deterministic steady state) Consider a deterministic model where A_t is constant and equal to 1 in each period (recall that 1 is the unconditional mean of the stochastic process for A_t in the stochastic model). We will derive the steady state of this deterministic model under the assumption that the financial constraint is binding in equilibrium. Let the superscript SS denotes steady-state values. In steady state the equilibrium conditions (16) and (27) give $q^{o,SS} = 1 - \delta$ and $q^{m,SS} = 1$. The law of motion for the capital stock (13) gives the steady-state condition

$$(1 - \theta \beta_C R^{SS}) K^{SS} = (1 - \gamma)(1 - \theta) R^{SS} K^{SS} + \gamma w^{SS} l_E, \quad (29)$$

and (28) gives

$$R^{SS} = \frac{\partial F(K^{SS}, 1)}{\partial K} + 1 - \delta. \quad (30)$$

Substituting (30) in (29) we obtain

$$K^{SS} = \left(\frac{\alpha(\theta \beta_C + (1 - \gamma)(1 - \theta)) + \gamma(1 - \alpha) l_E}{1 - (\theta \beta_C + (1 - \gamma)(1 - \theta))(1 - \delta)} \right)^{\frac{1}{1-\alpha}}, \quad (31)$$

and substituting back in (30) we get

$$R^{SS} = \alpha (K^{SS})^{\alpha-1} + 1 - \delta. \quad (32)$$

By assumption, the model parameters are such that $\beta R^{SS} < 1$. A necessary and sufficient

condition for this is that the model's parameters satisfy

$$\beta_E \left(\alpha \frac{1 - (\theta\beta_C + (1 - \gamma)(1 - \theta))(1 - \delta)}{\alpha(\theta\beta_C + (1 - \gamma)(1 - \theta)) + \gamma(1 - \alpha)l_E} + 1 - \delta \right) > 1. \quad (33)$$

Then, the following two inequalities also hold

$$\theta\beta_C R^S < 1, \quad \frac{(1 - \gamma)(1 - \theta)R^{SS}}{1 - \theta\beta_C R^{SS}} < 1. \quad (34)$$

To prove these inequalities, notice that there cannot be a steady state with a binding financial constraint and $K^{SS} = 0$. Otherwise, R^{SS} would go to infinity, violating $\beta_E R^{SS} < 1$. So K^{SS} must be strictly positive. Then, rearranging equation (29), it follows that

$$1 - \beta_C \theta R^{SS} - (1 - \gamma)(1 - \theta)R^{SS} > 0,$$

which implies both inequalities in (34).

The inequality $\beta R^{SS} < 1$ and the two inequalities in (34) correspond to conditions (6)-(8) in Proposition 3. Condition (10) holds immediately, given that $\beta_E < \beta_C$. This confirms that the entrepreneurs' optimal behavior is consistent with (13) (and (29)).

For completeness, we can use the recursive definition (9) to derive the steady state value of $\phi(X)$

$$\phi^{SS} = \frac{(1 - \theta)\beta_E R^{SS}}{1 - \theta\beta_C R^{SS}} \left(\gamma + (1 - \gamma)\phi^{SS} \right).$$

Rearranging this equation confirms that $\phi^{SS} > 1$.

Step 2. (Stability) Substituting (15), (28) and the lagged version of (14) into (13), we obtain the following second-order stochastic difference equation for K_t

$$K_{t+1} = \frac{(1 - \gamma)(1 - \theta) \left(A_t \frac{\partial F(K_t, 1)}{\partial K_t} - \frac{\partial G(K_{t+1}, K_t)}{\partial K_t} \right) K_t + \gamma A_t \frac{\partial F(K_t, 1)}{\partial L} l_E}{\frac{\partial G(K_{t+1}, K_t)}{\partial K_t} - \theta\beta_C \mathbb{E}_t \left[\left(A_{t+1} \frac{\partial F(K_{t+1}, 1)}{\partial K_{t+1}} - \frac{\partial G(K_{t+2}, K_{t+1})}{\partial K_{t+1}} \right) \right]}.$$

Linearizing this equation (under the functional assumptions made in the text) we get the following second order equation for $k_t = \ln K_t - \ln K^{SS}$,

$$\alpha_0 k_t + \alpha_1 k_{t+1} + \alpha_2 k_{t+2} = 0,$$

where

$$\begin{aligned} \alpha_0 &= \xi + \alpha(1 - \alpha)(\gamma l_E - (1 - \gamma)(1 - \theta))(K^{SS})^{\alpha-1} + (R^{SS} - \xi)(1 - \gamma)(1 - \theta), \\ \alpha_1 &= -\xi - 1 + \beta\theta R^{SS} - \beta\theta \left(\xi + \alpha(1 - \alpha)(K^{SS})^{\alpha-1} \right) + (1 - \gamma)(1 - \theta)\xi, \\ \alpha_2 &= \beta\theta\xi. \end{aligned}$$

Provided that

$$\alpha_1^2 - \alpha_0\alpha_2 > 0 \quad (35)$$

it is possible to show that the steady state derived in Step 1 is saddle-path stable. Then, given

sufficiently small shocks we can construct a stochastic steady state where K_t varies in a neighborhood of K^{SS} . This gives us an ergodic distribution for the state vector X , with bounded support. We can then establish the continuity of the function ϕ with respect to the parameters X and show that $\phi(X)$ is bounded in $[\underline{\phi}, \overline{\phi}]$. Since (6)-(8) hold in the deterministic steady state, a continuity argument shows that they hold in the stochastic steady state. Finally, $\overline{A} - \underline{A}$ can be set so as to ensure that the bounds for $\phi(X)$ satisfy

$$\beta_C \underline{\phi} > \beta_E \overline{\phi}.$$

This guarantees that condition (10) is also satisfied.

Notice that, by the arguments given, conditions (33) and (35) are sufficient to ensure that the steady state satisfies conditions (i) and (ii) in the hypothesis of the proposition.

B. The model with firm-level heterogeneity

Let w and q^o denote the constant values for the wage and the price of used capital. The (gross) return per unit of capital is now defined as:

$$R(A_{j,t}) \equiv \max_{\eta} \{A_{j,t} F(1, \eta) - w\eta + q^o\},$$

where η is the labor to capital ratio. The state variables for an individual entrepreneur are now $k_{j,t}$, $b_{j,t}$, and $A_{j,t}$. The entrepreneur's problem is characterized by the Bellman equation:

$$\begin{aligned} V(k, b, A) = & \max_{c^E, k', b'(\cdot), b'_L(\cdot)} c^E + \beta_E (1 - \gamma) \mathbb{E}[V(k', b'(\epsilon'), \Gamma(A, \epsilon'))] + \\ & + \beta_E \gamma \mathbb{E}[R(\Gamma(A, \epsilon')) k' - b'_L(\epsilon')] \end{aligned}$$

subject to

$$\begin{aligned} c^E + k' & \leq R(A) k - d, \\ b & = d + \beta_C ((1 - \gamma) \mathbb{E}[b'(\epsilon')] + \gamma \mathbb{E}[b'_L(\epsilon')]), \end{aligned}$$

$$\begin{aligned} b'(\epsilon') & \leq \theta R(\Gamma(A, \epsilon')) k', \\ b'_L(\epsilon') & \leq \theta R(\Gamma(A, \epsilon')) k'. \end{aligned}$$

The no-default constraints have been expressed as linear constraints, proceeding as we did in Proposition 3.

Now the marginal value of entrepreneurial wealth, ϕ , is a function of the individual productivity A and we have

$$\phi(A) = \frac{\beta_E (1 - \theta) \mathbb{E}[(\gamma + (1 - \gamma) \phi(\Gamma(A, \epsilon')) R(\Gamma(A, \epsilon')))]}{1 - \theta \beta_C \mathbb{E}[R(\Gamma(A, \epsilon'))]}.$$

The analogues to conditions (6)-(10) are now

$$\begin{aligned} \beta_E \mathbb{E}[R(\Gamma(A, \epsilon'))] & > 1, \\ \theta \beta_C \mathbb{E}[R(\Gamma(A, \epsilon'))] & < 1, \\ \frac{(1 - \gamma) (1 - \theta) \mathbb{E}[R(\Gamma(A, \epsilon'))]}{1 - \theta \beta_C \mathbb{E}[R(\Gamma(A, \epsilon'))]} & < 1, \end{aligned}$$

and

$$\phi(A) > \frac{\beta_E}{\beta_C} \phi(\Gamma(A, \epsilon')).$$

Under these conditions the optimal individual policy can be derived as in Proposition 3, and we obtain the following law of motion for the individual capital stock

$$k' = \frac{(1 - \theta) R(A)}{1 - \theta \beta_C \mathbb{E}[R(\Gamma(A, \epsilon'))]} k.$$

A newborn entrepreneur has initial wealth wl_E . Putting together these conditions, the distribution Φ and the law of motion $\Gamma(A, \epsilon')$, allows us to completely characterize the joint dynamics of k and A . Then, under appropriate assumptions, we obtain an ergodic joint distribution $J(A, k)$ and check that the wage rate w is consistent with the market clearing condition

$$\int [\eta(A) k] dJ(A, k) = 1,$$

where $\eta(A)$ is the optimal labor to capital ratio for a firm with productivity A .

Proceeding as in subsection 2.5, we can define the financial value of a continuing firm j :

$$p_{j,t} = \phi_{j,t} (R_{j,t} k_{j,t} - b_{j,t}) + b_{j,t} - d_{j,t}.$$

Substitute for $d_{j,t}$, using the budget constraint $d_{j,t} = R_{j,t} k_{j,t} - k_{j,t+1}$, and the law of motion for the capital stock

$$k_{j,t+1} = \frac{1}{1 - \theta \beta_C \mathbb{E}[R_{j,t+1}|A_{j,t}]} (R_{j,t} k_{j,t} - b_{j,t}),$$

to obtain

$$p_{j,t} = \left(\phi_{j,t} + \frac{\theta \beta_C \mathbb{E}[R_{j,t+1}|A_{j,t}]}{1 - \theta \beta_C \mathbb{E}[R_{j,t+1}|A_{j,t}]} \right) (R_{j,t} k_{j,t} - b_{j,t}).$$

Dividing both sides by $k_{j,t+1}$ and using the recursive property of $\phi_{j,t}$ gives the following expression for average q

$$q_{j,t} = \beta_E (1 - \theta) \mathbb{E}[(\gamma + (1 - \gamma) \phi_{j,t+1}) R_{j,t+1}|A_{j,t}] + \theta \beta_C \mathbb{E}[R_{j,t+1}|A_{j,t}].$$

For the investment rate notice that

$$IK_{j,t} = \frac{1}{k_{j,t}} \left(\frac{(1 - \theta) R_{j,t}}{1 - \theta \beta_C \mathbb{E}[R_{j,t+1}|A_{j,t}]} k_{j,t} - k_{j,t} \right),$$

which gives the expression in the text. For cash flow notice that $A_{j,t} F(k_{j,t}, l_{j,t}) - wl_{j,t} = R_{j,t} k_{j,t} - q^o k_{j,t}$.

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